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Director orientational instability in a planar flexoelectric nematic cell with easy axis gliding

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ABSTRACT

The orientational instability of a director in a planar flexoelectric nematic liquid crystal cell in a constant electric field oriented perpendicular to the cell substrates is investigated. The easy axis on the surface of one of the polymer substrates can change its orientation, which is due to the impact of the initial surface orientation, liquid crystal, and electric field. The influence of the latter leads to the reorientation of the elastic parts of the polymer molecules of the substrate, which is a consequence of the interaction of intrinsic or induced dipole moments with the electric field. The corresponding contribution of such an interaction to the surface free energy of the nematic is considered to be linear or quadratic in terms of the electric field strength **E**. It is established that the orientational instability of the director has a threshold in the case of quadratic effect of the electric field and is thresholdless if the effect is linear. The temporal behavior of the director after the application of voltage with the subsequent transition of the system to a stationary state and its return to the initial homogeneous state after switching off the voltage is studied. The characteristic turn-on/off times of the system and the time of reaching the stationary state are calculated and their dependence on the system parameters is investigated. The transmittance of the nematic liquid crystal cell is calculated for a normally incident light.

1. Introduction

Intensive research in liquid crystal (LC) physics in recent decades has contributed to the rapid development of liquid crystal display technologies. The industrial application of LCs is closely related to the possibility of relatively easy control of their orientation [1]. The latter significantly depends on the geometric shape of a substrate surface and the conditions for a director on it [2–5], namely, on an anchoring energy, the direction of an easy axis, etc.

It turns out that for certain conditions the characteristic time of director reorientation on the bounding surface of LC can significantly exceed the one of bulk deformations. This effect can be related to the gliding of the first monolayer of LC on the substrate surface, such as observed in lyotropic LC in a magnetic field [6] and thermotropic nematic liquid crystal (NLC) in the electric field [7]. Another reason is the possibility for the easy axis of a director on a polymer substrate to change its orientation in a low-frequency magnetic or electric field. Several mechanisms can explain such a gliding of the easy axis.

The easy axis gliding in a surface plane (azimuthal direction) of a polymer substrate of NLC cell in the presence of electric [8] and magnetic [9] fields, according to the authors, is due to the interaction of molecules of the LC with elastic fragments of polymer molecules. This mechanism was used for an explanation of the easy axis gliding in the presence of electric field also in the direction (polar) perpendicular to the substrate of NLC cell [10,11].

Another explanation for the reorientation of an easy axis is associated with adsorption of NLC molecules on the surface of a polymer substrate. The orienting effect of an external field, e.g. electric, leads to the rotation of the NLC molecules adsorbed on the surface. As a consequence, it causes the reorientation of an easy axis both in azimuthal [12–16] and polar [17] directions. The required external field and characteristic times of easy axis gliding are several orders of magnitude larger when an adsorption of NLC molecules on a substrate surface takes place. Due to the phenomenon of adsorption, the gliding of the easy axis in the azimuthal direction of the substrate surface, however, under the action of only the mechanical moment in the absence of external fields was experimentally studied in [18].

The change of an easy axis orientation on the surface of a polymer substrate in electric field can be caused by reorientation of the elastic parts of the polymer molecules due to the interaction of their intrinsic or induced dipole moments with the electric field

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[19]. Such a mechanism explained the induction of orientational anisotropy by electric field on the surface of the electrosensitive polymer in [20]. Within this model of easy axis gliding on the substrate surface, we investigated the effect of this phenomenon on the electric field-induced homeotropic-to-planar [21,22] and planar-to-planar [23,24] reorientations of a director in a NLC cell. However, in [21–24] the flexoelectric properties of NLC were not taken into account.

In this paper, the orientation instability of a director in a planar cell of a flexoelectric NLC in a constant electric field oriented perpendicular to the cell plane is theoretically studied. The direction of an easy axis can change on the surface of one of polymer substrates due to the impact of the initial surface orientation, liquid crystal and electric field. It is established that, depending on the nature of the influence of electric field on the movable easy axis, the orientation instability of a director can have a threshold behavior or either be thresholdless. The temporal behavior of a director field in the NLC bulk after switching on the electric field with the subsequent transition of the system to a stationary state and its return to the initial homogeneous state after switching off the voltage is considered. The influence of the system parameters, in particular, flexoelectric coefficients of the nematic on this behavior is studied. The time of the system transition to the stationary state and characteristic turn-on/off times are calculated and their dependence on the system parameters is investigated. The transmittance of the NLC cell is calculated for a normally incident light.

The conducted theoretical research expands the understanding of electrically induced reorientation of a NLC cell with electric field-changeable surface conditions on substrates.

2. NLC free energy and equations for the director

We consider a plane cell of flexoelectric NLC bounded by planes $z = 0$ and $z = L$, with the initial planar director orientation along the $0x$ axis. A constant potential difference U between cell substrates induces a constant electric field $\mathbf{E}$ along Oz in the bulk of NLC. The anchoring of NLC with the surface $z = 0$ of the lower substrate is taken to be infinitely strong. The easy axis $\mathbf{s}$ of director orientation on the surface $z = L$ of the upper polymer substrate can change its orientation in the direction perpendicular to the substrate due to the influence of the electric field [10,11]. We investigate the director field reorientation with the impact of the easy axis deviation on it.

The free energy of the NLC cell can be written in the form

$$F = F_{el} + F_E + F_S, \tag{1}$$

$$F_{el} = \frac{1}{2}\int_V \left\{K_1(\mathrm{div}\,\mathbf{n})^2 + K_2(\mathbf{n}\cdot\mathrm{rot}\,\mathbf{n})^2 + K_3[\mathbf{n}\times\mathrm{rot}\,\mathbf{n}]^2\right\} dV,$$

$$F_E = \int_V \left(-\frac{\varepsilon_0}{2}\mathbf{E}\hat{\varepsilon}\mathbf{E} - \mathbf{PE}\right) dV,$$

$$F_S = -\frac{W}{2}\int_S (\mathbf{sn})^2\, dS - \frac{W_0}{2}\int_S (\mathbf{ss}_0)^2\, dS + F_{SE}, \quad F_{SE} = -\frac{\alpha}{m}\int_S (\mathbf{sE})^m\, dS,$$

where $W > 0, W_0 > 0, S = S_{z=L}, \alpha > 0, m = 1$ or 2.

Here F_{el} is the elastic energy of a NLC, F_E encompasses the anisotropic and flexoelectric contributions to the free energy from the interaction of a NLC with the electric field, F_S denotes the surface free energy of a NLC, K_1, K_2, K_3 are the NLC elastic constants, $\mathbf{n}$ is a director, $\hat{\varepsilon} = \varepsilon_\perp \hat{\mathbf{1}} + \varepsilon_a \mathbf{n}\otimes\mathbf{n}$, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp > 0$ are tensor and anisotropy of the static permittivity of NLC, $\mathbf{P} = e_1\mathbf{n}\,\mathrm{div}\mathbf{n} - e_3[\mathbf{n}\times\mathrm{rot}\mathbf{n}], e_1, e_3$ denote the vector of flexoelectric polarization and flexoelectric coefficients of the NLC, W is an anchoring energy between director $\mathbf{n}$ and the easy axis $\mathbf{s}$ on the surface $z = L, W_0$ is an anchoring energy of the easy axis $\mathbf{s}$ with an initial surface orientation $\mathbf{s}_0$ along the Ox direction, α is the parameter of a coupling between the electric field $\mathbf{E}$ and the gliding easy axis $\mathbf{s}$.

The surface free energy F_S in (1) contains the contributions from interactions of the gliding easy axis $\mathbf{s}$ on the upper polymer substrate with the initial surface orientation $\mathbf{s}_0$ and the director $\mathbf{n}$. Both are written in the form of Rapini potential. F_S also takes into account the term F_{SE} describing the impact of electric field $\mathbf{E}$ on the orientation of the gliding easy axis $\mathbf{s}$, which arises due to the action of electric field on dipole moments of the elastic fragments of the polymer molecules. The corresponding contribution F_{SE} is considered to be linear ($m = 1$ in F_{SE}) or quadratic ($m = 2$) in electric field $\mathbf{E}$, depending on whether the electric dipole moments of the elastic fragments are intrinsic or induced by the field.

We consider plane deformations of the director field of NLC [10,11], lying in the xOz plane. Due to homogeneity of the system in the Oy direction, director in the bulk and the gliding easy axis on the surface $z = L$ have the following form

$$\mathbf{n} = (\cos\theta, 0, \sin\theta), \quad \mathbf{s} = (\cos\psi, 0, \sin\psi), \tag{2}$$

where $\theta(z,t), \psi(t)$ are the angles of deviation of the director and the gliding easy axis from their initial orientation along Ox.

The equations for director must be solved together with the equations for an electric field in the nematic bulk. As follows from the equation $\mathrm{rot}\,\mathbf{E} = 0$, the electric field in the bulk has a form $\mathbf{E} = (0, 0, E_z(z))$. Then, according to the equation $\mathrm{div}\,\mathbf{D} = 0$ a D_z component of the electric induction is constant and equals

$$D_z = \varepsilon_0\varepsilon_{zz}E_z + \frac{e}{2}\theta\prime_z \sin 2\theta, \tag{3}$$

where $\varepsilon_{zz} = \varepsilon_\perp + \varepsilon_a \sin^2\theta, e = e_1 + e_3$. Here and further, the primes denote the derivatives with respect to the corresponding arguments. Taking (3) into account, the potential difference between the cell substrates is equal to

$$U = \int_0^L E_z\, dz = D_z \int_0^L \frac{dz}{\varepsilon_0\varepsilon_{zz}} - \frac{e}{2\varepsilon_0\varepsilon_a}\ln\left(1 + \frac{\varepsilon_a}{\varepsilon_\perp}\sin^2\theta_L\right), \tag{4}$$

where $\theta_L = \theta(z = L)$ is the angle of director deviation on the surface of the upper substrate.

The free energy per unit area of the cell surface (1) takes the form

$$F = \frac{1}{2}\int_0^L \left[\left(K_1\sin^2\theta + K_3\cos^2\theta\right)\theta\prime_z^2 - \frac{\varepsilon_0\varepsilon_{zz}}{2}E_z^2 - \frac{e}{2}E_z\theta\prime_z\sin 2\theta\right]dz - \\ - \frac{W}{2}\cos^2(\theta_L - \psi) - \frac{W_0}{2}\cos^2\psi - \frac{\alpha}{m}(E_z\sin\psi)^m_{z=L}, \tag{5}$$

where the E_z component is determined by (3).

Minimizing the free energy (5) with respect to angles θ and ψ, we obtain the equation

$$\left(K_1\cos^2\theta + K_3\sin^2\theta\right)\theta\prime\prime_{zz} + \frac{1}{2}(K_3 - K_1)\theta\prime_z^2\sin 2\theta + \frac{\varepsilon_a D_z^2 \sin 2\theta}{2\varepsilon_0\varepsilon_{zz}^2} - \\ - \frac{e^2}{4\varepsilon_0\varepsilon_{zz}}\left(\frac{\varepsilon_a}{2\varepsilon_{zz}}\theta\prime_z^2\sin^3 2\theta - \theta\prime_z^2\sin 4\theta - \theta\prime\prime_{zz}\sin^2 2\theta\right) = \eta_v\theta\prime_t \tag{6}$$

and the boundary conditions

$$\theta|_{z=0} = 0, \tag{7}$$

$$\left[\left(K_1\cos^2\theta + K_3\sin^2\theta\right)\theta\prime_z - \frac{e}{2}E_z\sin 2\theta + \frac{W}{2}\sin 2(\theta - \psi)\right]_{z=L} = 0, \tag{8}$$

$$\left[\frac{W}{2}\sin 2(\theta-\psi)-\frac{W_0}{2}\sin 2\psi+\alpha E_z^m\cos\psi\sin^{m-1}\psi\right]_{z=L}=\eta_s\psi\prime_t, \quad (9)$$

where η_v, η_s are the coefficients of the NLC volume viscosity and the viscosity of the gliding easy axis, respectively, $m = 1$ or 2. Eq. (6) determines the temporal behavior of director in the bulk. The torque of viscosity forces in the right side of Eq. (6) is balanced by the sum of the torques of elastic forces and an electric field [1,25,26]. The sum is defined by variation $-\delta F/\delta\theta$. Eq. (9) describes the reorientation of the gliding easy axis. In the right side of the equation the torque of viscosity forces [14] is balanced by the sum of elastic torques from subsurface layer of NLC (proportional to W) and initial surface orientation (proportional to W_0) and electric field torque. The sum is defined by variation $-\delta F/\delta\psi$.

For the case of an arbitrary value of the applied voltage U the deviation angles of the director $\theta(z,t)$ and of the easy axis $\psi(t)$ can be found only numerically solving the Eq. (6) with the boundary conditions (7)–(9).

3. Linear coupling between the easy axis and the electric field

Let the elastic fragments of the molecules of the upper polymer substrate have intrinsic dipole moments. The easy axis **s** corresponds to the predominant orientation of the director on the $z = L$ surface. In this case, the contribution from the interaction between the easy axis **s** and the electric field **E** to the surface free energy is linear in the electric field ($m = 1$ in F_{SE}(1)). It should be noted that the α coefficient in F_{SE} has the meaning of polarization per unit area of the substrate.

After the voltage U is switched on and until the system reaches the stationary state, the temporal behavior of the director and the gliding easy axis is determined by the applied voltage and the NLC cell parameters. Fig. 1 illustrates the angle θ of the director deviation within of the cell thickness, calculated at different moments of time during the transition of the system to the stationary state. The calculations were performed using the following typical NLC parameter values[27]: $K_1 = 6$pN, $K_3 = 10$pN, L = 10 μm at the physical assumption $W_0 \gg W$ ($w_0 \gg w$ for dimensionless anchoring energies $w_0 = W_0L/K_1$ and $w = WL/K_1$) due to the stronger anchoring of the easy axis with the initial surface orientation than director with the easy axis [16]. According to calculations, at the first moments of time after switching on the voltage the gliding easy axis at $z = L$ deviates faster than the director of the near-surface layer. The latter follows the rotation of the easy axis, so that

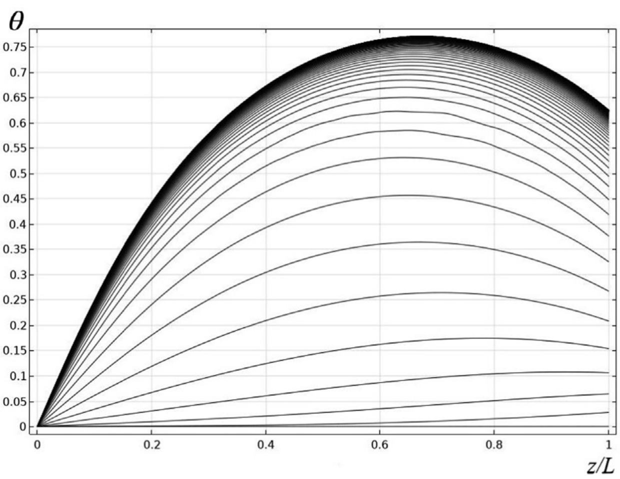

Fig. 1. Temporal behavior of the director angle within the cell for the first 30 s after the turning on the voltage with a time step of 1 s. $U = 1$ V, $e = 17$ pC/m, $w = 10, w_0 = 100, \alpha = 266$ pC/m, $\eta_v = 0.1$ Pa·s [1], $\eta_s = 10^{-4}$ Pa·m·s [14].

$\psi \geqslant \theta_L$. The dependence of the director angle θ on the coordinate z is practically linear. This behavior of the system is due to the influence of the electric field on the gliding easy axis. However, a further increase in the value of the angle ψ of the easy axis slows down due to its interaction with the initial surface orientation. The easy axis starts to lag behind the director in the near-surface layer, $\psi \leqslant \theta_L$. The largest deformations of the director field are shifted from the surface into the bulk of NLC. For small voltage U we can implement the approximation of small angles and describe the behavior of the system by the linearized with respect to the θ and ψ Eq. (6)

$$\theta\prime\prime_{\xi\xi}+u^2\theta=\theta\prime_\tau \quad (10)$$

and boundary conditions (7)–(9)

$$\theta|_{\xi=0}=0, \quad (11)$$

$$[\theta\prime_\xi+w(\theta-\psi)-\tilde{e}u\theta]_{\xi=1}=0, \quad (12)$$

$$[w(\theta-\psi)-w_0\psi+\tilde{\alpha}u]_{\xi=1}=\gamma\psi\prime_\tau. \quad (13)$$

Here we use the dimensionless coordinate $\xi = z/L$, time $\tau = tK_1/(\eta_v L^2)$, relative viscosity $\gamma = \eta_s/(\eta_v L)$, coupling parameter $\tilde{\alpha} = \alpha/\sqrt{\varepsilon_0\varepsilon_a K_1}$, flexoelectric constant $\tilde{e} = e/\sqrt{\varepsilon_0\varepsilon_a K_1}$, voltage $u = \pi U/U_{th}^\infty$, where $U_{th}^\infty = \pi\sqrt{K_1/\varepsilon_0\varepsilon_a}$ is the Fréedericksz transition at $w \to \infty$ without the easy axis gliding.

Because of cumbersomeness of obtaining the solution of the Eq. (10) which satisfies the boundary conditions (11)–(13), the detailed derivation is given in the Appendix. In approximation of $\gamma \gg 1$ [14], accounting only for the first-order terms in $1/\gamma$, the temporal dependencies of the angles of the director and the gliding easy axis reorientation has the following form:

$$\begin{aligned}\theta(\xi,\tau) &\approx \xi\frac{\tilde{\alpha}uw}{(w+w_0)(1-\tilde{e}u)+ww_0}\left(1-e^{-\tau/\tau_{on}}\right),\\ \psi(\tau) &\approx \frac{\tilde{\alpha}u\tau_{on}}{\gamma}\left(1-e^{-\tau/\tau_{on}}\right),\end{aligned} \quad (14)$$

where

$$\tau_{on}=\frac{\gamma(1+w-\tilde{e}u)}{(w+w_0)(1-\tilde{e}u)+ww_0} \quad (15)$$

has the meaning of the dimentionless characteristic turn-on time of the system and it was implemented here similarly to linear analysis of director dynamics given in [1]. The solutions show that the orientational instability of the NLC has no threshold. At arbitrary values of the applied voltage, there are increasing in time deviations of the director and the easy axis in the system.

The characteristic turn-on time τ_{on} increases with increasing sum e of the flexoelectric coefficients of NLC, relative viscosity coefficient γ and with decreasing anchoring energies w and w_0. Increasing the applied voltage u leads to an increase of τ_{on} in the case of positive values of e and its decrease for $e < 0$, respectively. The characteristic turn-on time τ_{on} practically does not depend on the coupling parameter α.

Fig. 2a shows the dependencies of the time τ_{sat} of the system entering the stationary state on the anchoring energy w, calculated at different values of the coupling parameter α. It should be noticed that the values of time parameters τ_{sat} and τ_{on} do not match. Here, τ_{sat} determines the duration of the transition process after turning on voltage till the establishing the stationary state in a NLC cell in general case of arbitrary deformations of director field. At the same time, τ_{on} is defined in the solution (14) of equation and boundary conditions (10)–(13) linearized with respect to angles θ and ψ. As one can see, with the exception of the region of weak coupling $w \lesssim 1$, the time τ_{sat} increases monotonically with increasing w and in the region of strong anchoring $w \gtrsim 10$ it reaches a constant value depending on values of the parameter α. This behavior τ_{sat}

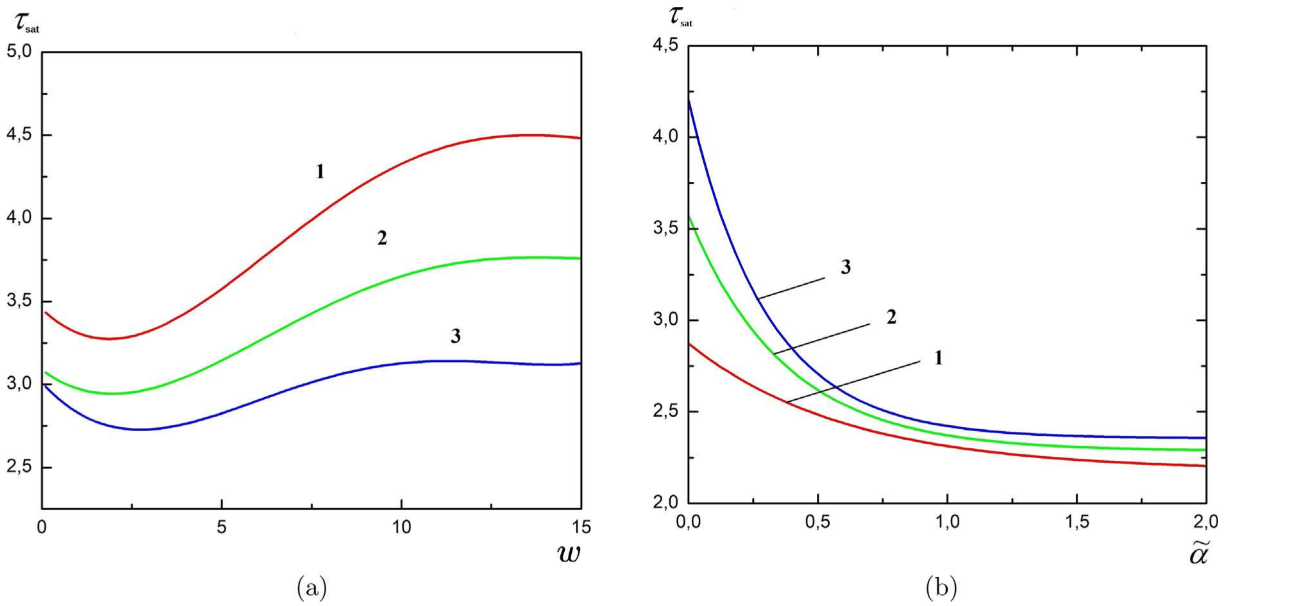

Fig. 2. Dependence of the characteristic time τ_{sat} of reaching the stationary state on the anchoring energy w (a) and the coupling parameter $\tilde{\alpha}$ (b). $U = 0.72$ V. (a) $\tilde{\alpha} = 0.1$ (1), 0.5 (2), 1 (3); (b) $w = 1$ (1), 5 (2), 10 (3).

is fully consistent with the dependence $\tau_{on}(w)$ (15) in the range of $w \gg 1$. It should be noted that with increasing anchoring energy, the impact of the easy axis gliding on the dynamics of the director becomes more noticeable, namely, the time required for the system to reach a stationary state increases. On the other hand, when the anchoring is weak, the impact of the gliding on the director reorientation decreases and, consequently, the system reaches the stationary state faster. In the range of $w \sim 1$ a slight decrease of τ_{sat} is observed. This is a consequence of the non-monotonic temporal behavior of the residual deformations of the director field after its relatively rapid reorientation by the electric field. It is caused by the inertia of the gliding easy axis at a relatively large value of the surface viscosity $\eta_s \gg \eta_v L$ and relatively weak anchoring of director with the easy axis.

As the coupling parameter α increase, the transition of the system to the stationary state accelerates for all values of the anchoring energy w (see Fig. 2b). This is due to the increasing impact of the electric field on the easy axis. Note that increasing the relative viscosity γ of the gliding easy axis slows down the entry of the system into a stationary state.

In the stationary state, the deformations of the director field, as calculations show, significantly depend on the sum e of the flexoelectric coefficients of NLC. The calculated distributions of the angle θ of the director deviation within the cell at different values of the sum e are shown in Fig. 3a. In the stationary state, the largest deformations of the director field can occur both in the bulk of NLC as well as near the surface with the gliding easy axis, which depends on the value of e. This behavior of the deformations in the electric field is due to the combined effect of the gliding easy axis and flexopolarization on the director. In particular, at negative values of e, the influence of flexopolarization and the gliding easy axis on the reorientation of the director is mutually opposite. That is, the presence of flexopolarization leads to a decrease of the deformations of the director field, while the influence of the electric field on the movable easy axis on the contrary increases them. In general, a decrease of the parameter e in the region of its negative values leads to a decrease of the director field deformations in comparison with the case of NLC without flexoelectric properties. In this case, the largest deviations of the director angle θ always occur in the volume of NLC, so that $\theta_L \geqslant \psi$. At positive values of e, the flexopolarization enhances the effect of the gliding easy axis on the orientational instability increasing the deformation of the director field. Thus, an increase of e towards its positive values leads to an increase of the deformations in comparison to the case of $e = 0$. Accordingly, the position of the maximum deviations of the angle θ of the director gradually shifts from the volume of NLC towards the surface with a gliding easy axis. If the largest deformations of the director field is in the near-surface layer of NLC, then $\theta_L \leqslant \psi$.

Fig. 3b shows the stationary distributions of the angle θ of the deviation of the director across the cell at several values of the coupling parameter α. As the parameter α increases, the director field deformations increase. This behavior of the system is expected and is due to the increasing influence of the electric field on both the director of the liquid crystal and the gliding easy axis. In general, an increase in the voltage U and a decrease of the anchoring energy w leads to an increase of the director field deformations in the bulk.

Further, we calculate the transmittance of the NLC cell for a plane monochromatic light wave with frequency ω and intensity I_0 in the case of its normal incidence on the cell substrates. We assume that the NLC cell is in a stationary state, and the light wave does not change the spatial distribution of the director field. According to the Malus's law, the transmission coefficient of light passing through a cell placed between crossed polarizers has the form

$$T = I/I_0 = \sin 2\beta \sin^2(\Delta\Phi/2), \tag{16}$$

where I_0 is the intensity of incident light, β is the angle between the plane of polarization of the polarizer and the plane of reorientation of the director, $\Delta\Phi$ is the phase difference between ordinary and extraordinary rays at the outlet of the NLC cell [27]:

$$\Delta\Phi = \frac{2\pi}{\lambda} \int_0^L [n_e(z) - n_o] dz, \tag{17}$$

where $n_o = \sqrt{\varepsilon_\perp^\omega}$ and n_e denotes a refractive indexes for ordinary and extraordinary rays:

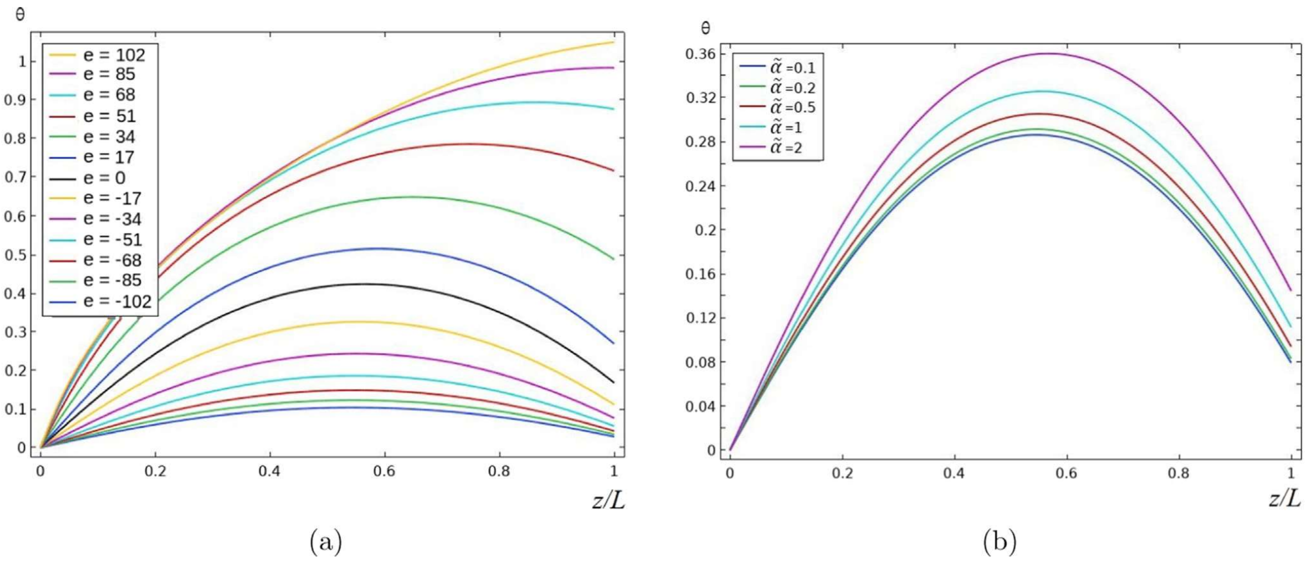

Fig. 3. Stationary dependencies of the director angle θ within the cell. $U = 0.72$ V, $w = 10, w_0 = 100$. (a) $\tilde{\alpha} = 1$ at different values of e[pC/m]; (b) $e = -17$ pC/m for different values of $\tilde{\alpha}$.

$$n_e(z) = \frac{\sqrt{\varepsilon_\perp^\omega}\sqrt{\varepsilon_\parallel^\omega}}{\sqrt{\varepsilon_\perp^\omega \cos^2\theta(z) + \varepsilon_\parallel^\omega \sin^2\theta(z)}}. \tag{18}$$

Here $\varepsilon_\perp^\omega$ and $\varepsilon_\parallel^\omega$ are permittivities of NLC in perpendicular and parallel direction to an optical axis, respectively, for a light wave with frequency ω [27].

The dependence of the light transmittance T on the value of the applied voltage U is shown in Fig. 4a at several values of the coupling parameter α between the electric field and the gliding easy axis. As follows from the above dependencies, the presence of coupling between the electric field and the gliding easy axis significantly changes the nature of the orientational instability of the director in the NLC cell volume. Thus, in the case of the absence of the influence of the electric field on the gliding easy axis (parameter $\alpha = 0$), the orientational instability has a threshold. Whereas the presence of such an influence ($\alpha \neq 0$) makes the orientational instability thresholdless. In general, the ability of the easy axis to glide accelerates the transition of the system to a stationary state.

Fig. 4b shows the dependence of the light transmittance T on the value of the applied voltage U for different types of boundary conditions for the director on the upper substrate of the cell. Note that after the system transitions from the initial homogeneous state to inhomogeneous under the influence of the electric field **E** the possibility of easy axis gliding practically does not change the qualitative nature of the dependence $T(U)$.

Further, we consider the relaxation of the system from the stationary state, maintained by the voltage U, to the initial homogeneous state after the voltage is turned off. In the approximation of small angles θ and ψ the relaxation of the system can be

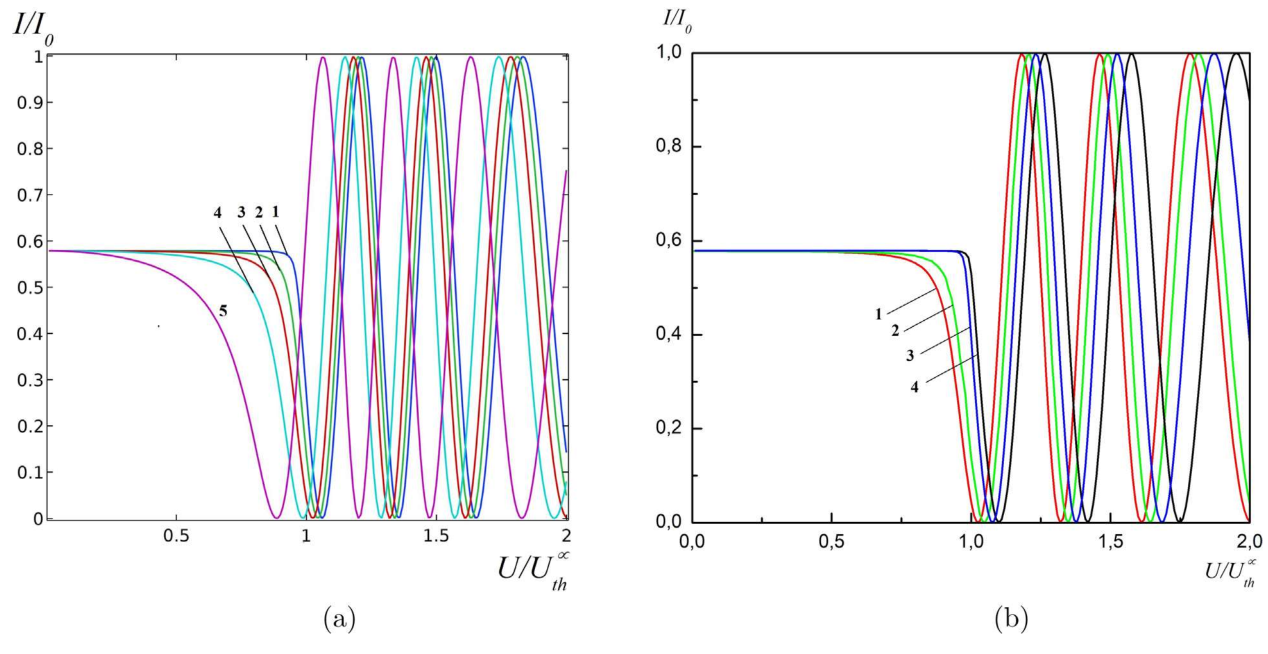

Fig. 4. Light transmittance T of the NLC cell as a function of applied voltage U for the light wave with $\lambda = 532$ nm [28]. (a) $w = 10, w_0 = 100$. $\tilde{\alpha} = 0.1$ (1), 0.5 (2), 1 (3), 2 (4), 5 (5); (b) $\tilde{\alpha} = 1, w = 50, w_0 = 100$: linear (1) and quadratic (2) coupling between the electric field and the easy axis; (3) $\tilde{\alpha} = 0, w = 50$; (4) $\tilde{\alpha} = 0, w \to \infty$.

described by linearized Eqs. (10) and boundary conditions (11)–(13) with $u = 0$. At the approximation $\tau \to +\infty$ solution of the linearized problem gives the values of the angles of the director and the gliding easy axis, respectively

$$\begin{aligned} \theta(\xi,\tau) = & \sum_{n=1}^{\infty} A_n e^{-\varkappa_n^2\tau} \sin \varkappa_n \xi, \\ \psi(\tau) = & \sum_{n=1}^{\infty} \frac{A_n \varkappa_n}{\gamma\varkappa_n^2 - w_0} e^{-\varkappa_n^2\tau} \cos \varkappa_n \xi, \end{aligned} \quad (19)$$

where A_n are the integration constants, $\varkappa_n$ denote the positive roots of the equation

$$\varkappa = \frac{\varkappa}{\gamma\varkappa^2 - w_0} - \frac{\varkappa}{w}. \quad (20)$$

As follows from (19), the dimentionless characteristic turn-off time of the system is determined by the slowest decaying mode with $n = 1$ and is equal to $\tau_{off} = \varkappa_1^{-2}$.

Evidently, the turn-off time τ_{off} does not depend on the values of the coupling parameter α and the flexoelectric coefficients of NLC. However, the possibility of the easy axis gliding leads to an increase of the turn-off time τ_{off} compared to the absence of such gliding. The value of τ_{off} increases with decreasing anchoring energies w and w_0. This is due to the weakening of the interaction of the easy axis with the director and the initial surface orientation, respectively. Increasing the values of the viscosity parameter γ leads to an increase of relaxation time τ_{off}.

4. Quadratic coupling between the easy axis and the electric field

Let the dipole moments of the elastic parts of the molecules of the upper polymer substrate of the cell be induced by the electric field. Then, the contribution to the surface free energy F_S of NLC from the interaction of the gliding easy axis on the substrate with the electric field is quadratic in $\mathbf{E}$ ($m = 2$ in F_{SE} (1)). Minimization of free energy (5) with respect to the θ and ψ angles leads to the Eq. (6) and boundary conditions (7)–(9), in which $m = 2$. In the approximation of small θ and ψ angles the behavior of the system is described by the linearized Eq. (10), boundary conditions (11), (12) and the condition

$$\left[w(\theta-\psi) - w_0\psi + \tilde{\alpha}u^2\psi\right]_{\xi=1} = \gamma\psi\prime_\tau, \quad \text{where} \quad \tilde{\alpha} = \alpha/(\varepsilon_0\varepsilon_a L). \quad (21)$$

Requiring the solution of the Eq. (10) to satisfy the boundary conditions (11), (12) and (21), we obtain the following values of the angles of the director and the gliding easy axis

$$\theta(\xi,\tau) = \sum_{n=1}^{\infty} A_n \sin(\lambda_n\xi) e^{\Gamma_n\tau}, \quad (22)$$

$$\psi(\tau) = \sum_{n=1}^{\infty} \frac{A_n}{w}(\lambda_n \cos\lambda_n + (w - \tilde{e}u)\sin\lambda_n) e^{\Gamma_n\tau}, \quad (23)$$

where A_n are the integration constants, $\Gamma_n = u^2 - \lambda_n^2$, λ_n denote the positive roots of the equation

$$\lambda = \frac{\lambda(\gamma(u^2-\lambda^2) + w + w_0 - \tilde{\alpha}u^2)}{w^2 - (w - \tilde{e}u)(\gamma(u^2-\lambda^2) + w + w_0 - \tilde{\alpha}u^2)}. \quad (24)$$

As follows from the solutions (22) and (23), if at least one of the values $\Gamma_n > 0$, then the spatial perturbations of the director field of $\sin\lambda_n\xi$ type grow exponentially in time in the bulk of the nematic. Orientational instability of NLC has a threshold. The value of the threshold is determined by the condition $\Gamma_1 = 0$ and is corresponding to the smallest positive solution of the equation

$$u = \frac{u(w + w_0 - \tilde{\alpha}u^2)}{w^2 + (\tilde{e}u - w)(w + w_0 - \tilde{\alpha}u^2)}. \quad (25)$$

The dependence of the threshold U_{th} of Fréedericksz transition on the anchoring energy w, found from the Eq. (25), is shown in Fig. 5a for different values of the sum e of flexoelectric coefficients of NLC. Generally, the threshold U_{th} increases with increasing w, and in the case of strong anchoring $w \gtrsim 10$ tends to a constant value. The latter is achieved faster by decreasing the parameter e. For negative $e < 0$, the threshold U_{th} weakly depends on the changes of w starting from $w \gtrsim 5$. Positive value of e leads to a decrease of the threshold U_{th}, while negative value of e leads, on the contrary, to its increase compared to the case $e = 0$ of the absence of flexoelectric properties of NLC (see Fig. 5a). In general, increasing of e leads to a decrease of U_{th} for arbitrary values of the coupling parameter α and the anchoring energy w. Note that in the approximation of $w_0 \gg w$ the threshold U_{th} weakly depends on the coupling parameter α.

Fig. 5b shows the dependence of U_{th} at the sum of flexoelectric coefficients e of NLC for several values of the anchoring energy w. In the case of absence of flexoelectric properties of NLC ($e = 0$) and for $w \approx 0$, the threshold U_{th} approaches its value $0.5U_{th}^{\infty}$. The latter corresponds to the absence of anchoring of NLC with the $z = L$ surface of the upper substrate. Note that the threshold U_{th} is more sensitive to changes of the value of the anchoring energy w at positive values of the sum of flexoelectric coefficients e in comparison to the case of $e < 0$.

In the case of voltage U slightly exceeding the threshold value U_{th}, the values of the angles θ and ψ can be considered small and are determined by the strongest mode with $n = 1$ in the expressions (22) and (23) respectively. Higher modes can be neglected due to their smallness. This allows us to determine the characteristic turn-on time of the system

$$\tau_{on} = \frac{1}{(1-\sigma)(u^2 - u_{th}^2)}, \quad (26)$$

where

$$\begin{aligned} \sigma &= \frac{Au_{th}+C}{B_3u_{th}^3 + B_2u_{th}^2 + B_1u_{th} + C}, \\ A &= 2(\rho_0 - 1)^2(\gamma - \tilde{\alpha}), \quad C = -\tilde{e}(\rho\rho_0 - 1)^2, \\ B_1 &= (\rho^2 - \tilde{\alpha}\rho + \tilde{e}^2\rho^2 + 2\gamma)\rho_0^2 - (2\tilde{e}^2\rho + 2\rho + 4\gamma \\ &\quad -\tilde{\alpha})\rho_0 + 2\gamma + \tilde{e}^2 + 1, \\ B_2 &= 2\tilde{e}\tilde{\alpha}\rho_0(\rho\rho_0 - 1), \quad B_3 = \tilde{\alpha}^2\rho_0^2, \\ \rho &= 1 - \tilde{\alpha}u_{th}^2/w, \quad \rho_0 = 1 - \tilde{\alpha}u_{th}^2/w_0. \end{aligned}$$

In contrast to the case of linear coupling between the electric field and the gliding easy axis, the characteristic turn-on time τ_{on} decreases with increasing of the applied voltage u (where $u > u_{th}$), the relative viscosity coefficient γ, the sum e of the flexoelectric coefficients and with decreasing of the anchoring energy w_0. Increasing the values of w leads to an increase of the τ_{on} in the case of $e > 0$ and its decrease if $e < 0$. Note that when the voltage u approaches the threshold value u_{th} the characteristic times τ_{on} and τ_{sat} of the transition to a stationary state increase significantly because $\tau_{on} \sim (u^2 - u_{th}^2)^{-1}$.

Calculations show that the temporal behavior of the angles θ of the director and ψ of the gliding easy axis after the voltage is switched on with the subsequent transition of the system to a stationary state is qualitatively similar to the case of linear electric field coupling with the gliding easy axis (see Fig. 1). Note that the coupling of the easy axis with electric field does not change the character of the dependence of the light transmittance coeffi-

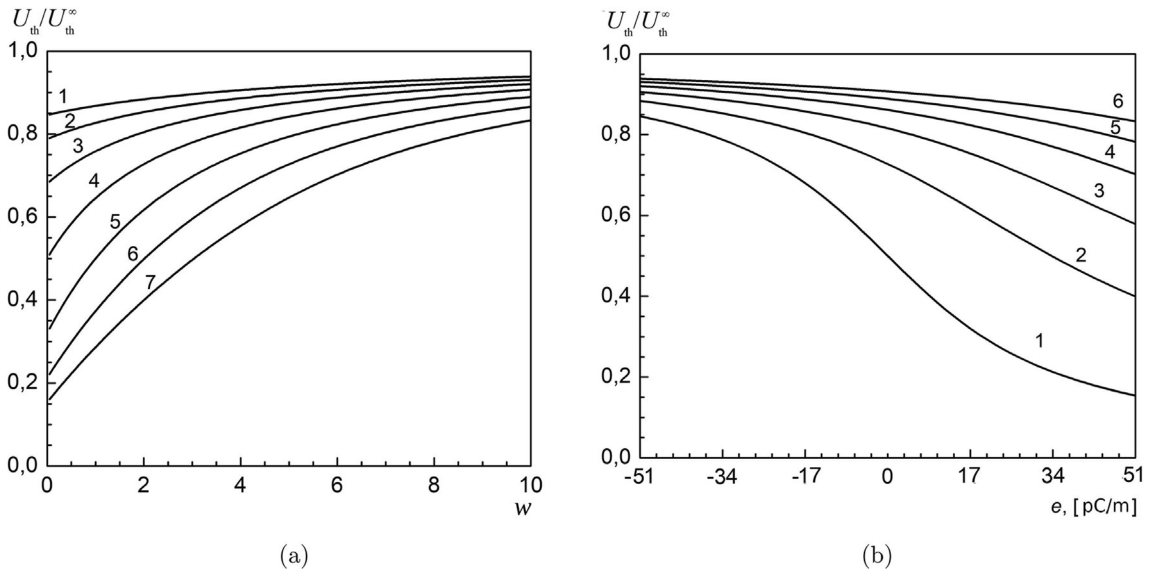

Fig. 5. Dependencies of the threshold U_{th} of the Fréedericksz transition on the anchoring energy w (a) and the sum e of flexocoefficients of NLC (b). $w_0 = 200, \tilde{\alpha} = 1$. (a) $e = -51$ (1), -34 (2), -17 (3), 0 (4), 17 (5), 34 (6), 51 (7)pC/m; (b) $w = 0$ (1), 2 (2), 4 (3), 6 (4), 8 (5), 10 (6).

cient T on the voltage U after the system transitions from the initial homogeneous to the inhomogeneous state (see Fig. 4b).

The relaxation of the system from the stationary state to the initial homogeneous state after the voltage is switched off evidently does not depend on the nature of the coupling of the electric field and the gliding easy axis. The characteristic switch-off time τ_{off} is equal to that of the linear coupling case.

5. Conclusions

The induced by the electric field reorientation of the director from the planar state to the homeotropic state in the cell of the flexoelectric NLC was studied theoretically. The possibility of gliding of the easy axis on the surface of one of the cell substrates in the direction perpendicular to the substrate was taken into account. In the proposed model the easy axis is simultaneously affected by the liquid crystal, the electric field, and the initial surface orientation. The contribution from the influence of the electric field on the easy axis to the surface free energy density of the nematic is considered to be linear or quadratic in **E** according to whether the dipole moments of the elastic parts of the polymer substrate molecules are intrinsic or induced by the electric field.

In the considered geometry of the problem, the electric field simultaneously destabilizes the initial orientation of both the director and the gliding easy axis. Under the linear in **E** coupling of the electric field with the easy axis, the orientational instability of NLC is thresholdless. After switching on the voltage, the temporal behavior of the system is determined by the applied voltage and physical properties of NLC, flexoelectric in particular. The value of the characteristic turn-on time τ_{on} decreases with decreasing sum e of flexoelectric coefficients of NLC, relative viscosity coefficient γ and with increasing values of anchoring energies w and w_0. An increase of the applied voltage U leads to an increase of the turn-on time τ_{on} in the case of the positive e and to a decrease in τ_{on} for $e < 0$, respectively.

In the stationary state, the largest deviations of the director can be both in NLC bulk and near the surface with a gliding easy axis, depending on the sum e of the flexoelectric coefficients. Decreasing e in the case of its negative values leads to a decrease of the deformations of the director field in comparison with the case of the absence of flexoelectric properties of NLC. In this case, the largest deformations of the director field are achieved in the volume of NLC. However, increasing the parameter e in the case of its positive values leads to an increase of the deformations of the director field in comparison with the case of $e = 0$. As a result, the maximum deviations of the director angle are shifted toward the surface with a gliding easy axis.

If the coupling between the electric field and the gliding easy axis is quadratic in the field strength **E**, then the orientational instability of the NLC has a threshold. Compared with the absence of flexoelectric properties of NLC, positive values of the flexoelectric coefficient e lead to a decrease in the value of the threshold U_{th}, and negative, on the contrary, to its increase. In general, an increase of e leads to a decrease of the threshold U_{th} for arbitrary values of the coupling parameter α and the anchoring energy w. In this case, the characteristic turn-on time τ_{on} decreases with increasing the applied voltage U (where $U > U_{th}$), the relative viscosity coefficient γ, flexoelectric parameter e and with decreasing value of anchoring energy w_0. Note that increasing anchoring energy w leads to an increase of the turn-on time τ_{on} in the case of $e > 0$ and its decrease for $e < 0$.

The relaxation of the system from a stationary state to the initial homogeneous state after switching off the voltage does not depend on the coupling type of the electric field and the movable easy axis. When the voltage is switched off, quite quickly, in comparison with the time of full relaxation, almost linear dependence of the director deviation angle on the coordinate across the cell is established. Throughout the relaxation, the largest deviations of the director occur near the surface with a gliding easy axis. In general, the director returns to the initial homogeneous state faster, pulling the easy axis behind.

For both coupling types between the easy axis and the electric field, the characteristic turn-off time τ_{off} of the system is the same and independent of the coupling parameter α and the sum e of flexoelectric coefficients of NLC. The τ_{off} decreases with decreasing value of viscosity parameter γ and with increasing anchoring energies w and w_0.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Excluding $\theta(\xi = 1, \tau)$ from (12), we introduce a function

$$f(\xi,\tau) = \theta(\xi,\tau) + \xi g(\tau), \text{ where } g(\tau) = (1 - \tilde{e}u/w)(\gamma\psi\prime_\tau - \tilde{\alpha}u + w_0\psi) - \tilde{e}u\psi. \quad (A.1)$$

The latter, as follows from (10)–(13), satisfies the equation

$$f\prime\prime_{\xi\xi} + u^2 f = f\prime_\tau - \xi(g\prime_\tau - u^2 g) \quad (A.2)$$

and homogeneous boundary conditions

$$f(\xi = 0, \tau) = f\prime_\xi(\xi = 1, \tau) = 0. \quad (A.3)$$

The solution of the Eq. (A.2), which satisfies the conditions (A.3), we write in the form

$$f(\xi,\tau) = \sum_{n=0}^{\infty} f_n(\tau) \sin[\pi(n + 1/2)\xi], \quad (A.4)$$

where $f_n(\tau)$ are the unknown coefficients. Substituting the expansion (A.4) in the Eq. (A.2) and using the linear independence of the functions $\sin[\pi(n+1/2)\xi]$ on $[0,1]$, we obtain the equation for the coefficients $f_n(\tau)$:

$$\frac{df_n}{d\tau} + \left(\pi^2(n+1/2)^2 - u^2\right)f_n = \frac{2(-1)^n}{\pi^2(n+1/2)^2}(g'_\tau - u^2 g), \quad \text{where} \quad n = 0, 1, 2, \ldots. \quad (A.5)$$

In a weak electric field $u \ll 1$ ($U \ll U_{th}^{\infty}$) we consider only the term with $n = 0$ in the expansion (A.4). The contribution of higher harmonics is neglected, considering them relatively small. Substitution of the explicit form of the function $f(\xi,\tau) = f_0(\tau)\sin(\pi\xi/2)$ in the condition (13) leads to the value

$$f_0(\tau) = g(\tau) + \psi(\tau) + (\gamma\psi\prime_\tau - \tilde{\alpha}u + w_0\psi)/w. \quad (A.6)$$

Substitution of (A.6) in the relation (A.5) for $n = 0$ leads to the equation

$$a\psi\prime\prime + b\psi\prime + c\psi = d \quad (A.7)$$

for the temporal dependence $\psi(\tau)$, which satisfies the initial conditions

$$\psi(0) = 0, \qquad \psi\prime_\tau(0) = \tilde{\alpha}u/\gamma. \quad (A.8)$$

Note that the latter condition (A.8) follows from (13) as a consequence of $\theta(\xi, \tau = 0) = 0$. The coefficients in (A.7) are

$$\begin{aligned} a &= 4\gamma\left[\pi^2 + (\pi^2 - 8)(w - \tilde{e}u)\right], \\ b &= \gamma\left[\pi^4(w - \tilde{e}u + 1) + 4\pi^2\left((\tilde{e}u - w - 1)(u^2 - w_0/\gamma) - w(\tilde{e}u - 1)/\gamma\right) - \right. \\ &\quad \left. -32\left(u^2(\tilde{e}u - w) - \tilde{e}u(w + w_0)/\gamma + ww_0\right)\right], \\ c &= \pi^4((w + w_0)(1 - \tilde{e}u) + ww_0) + 4\pi^2u^2((w + w_0)(\tilde{e}u - 1) - ww_0) - \\ &\quad - 32u^2(\tilde{e}u(w + w_0) - ww_0), \\ d &= \tilde{\alpha}u\left[\pi^4(w + 1 - \tilde{e}u) + 4\pi^2u^2(\tilde{e}u - 1 - w) - 32u^2(\tilde{e}u + w)\right]. \end{aligned}$$

Found from the Eq. (A.7), the time dependence of the deviation angle of the gliding easy axis at the surface $z = L$ takes the form:

$$\psi(\tau) = \frac{k_2 d/c - \tilde{\alpha}u/\gamma}{k_1 - k_2}e^{-k_1\tau} - \frac{k_1 d/c - \tilde{\alpha}u/\gamma}{k_1 - k_2}e^{-k_2\tau} + \frac{d}{c}, \quad (A.9)$$

where $k_1 = (b + \sqrt{b^2 - 4ac})/(2a), k_2 = (b - \sqrt{b^2 - 4ac})/(2a) > 0$. In the approximation of $\gamma \gg 1$ we have $k_1 \gg k_2$, since $k_1 \approx \frac{b}{2a}\left(2 + O\left(\frac{1}{\gamma}\right)\right)$, $k_2 \approx \frac{c}{b} = O\left(\frac{1}{\gamma}\right)$. Accordingly, the dependence $\psi(\tau)$ (A.9) takes the form

$$\psi(\tau) \approx \frac{d}{c}\left(1 - e^{-k_2\tau}\right). \quad (A.10)$$

Substitution of $\psi(\tau)$ (A.10) in (A.6) gives us the value of $f_0(\tau)$ and, accordingly, the function $f(\xi,\tau) = f_0(\tau)\sin(\pi\xi/2)$. Next, using (A.1) and taking into account the obtained values of $f(\xi,\tau)$ and $\psi(\tau)$ (A.10), we obtain the angle $\theta(\xi,\tau)$ of deviation of the director in the bulk.

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